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Informe Ejecutivo

TÍTULO: AFP-1.0: Instancias reales para problema de la asignación automática de frecuencias

RESUMEN: Este documento describe tres instancias reales del problema de la Asignación Automática de Frecuencias (AFP o *Automatic Frequency Planning*) que serán abordadas en el marco de DIRICOM. También se incluyen anto la formulación matemática utilizada para evaluar los planes de frecuencia como las mejores soluciones encontradas hasta el momento y el algoritmo utilizado en su caso.

OBJETIVOS:

1. Proporcionar una descripción detallada de las instancias que serán abordadas.
2. Dar la formulación matemática usada para evaluar estos datos procedentes de redes reales.
3. Informar sobre los costes de las mejores soluciones publicadas hasta el momento.

CONCLUSIONES:

1. Es de gran importancia evaluar las diferentes propuestas algoritmicas con instancias reales de los problemas. Así, en este informe se propocionan los datos de tres instancias reales, la forma en la que se usan dichos datos para evaluar planes de frecuencia y las mejores soluciones publicadas hasta el momento.

RELACIÓN CON ENTREGABLES:

PRE: —

CO: —

Málaga, December 2008

Executive Summary

TITLE: AFP-1.0: Real-world instances for the Automatic Frequency Planning problem

ABSTRACT: This deliverable describes the three real-world instances of the Automatic Frequency Planning problem (AFP) that will be addressed within the DIRICOM project. Both the mathematical formulation used to evaluate the frequency plans and the best known solutions found so far in the literature are also included.

GOALS:

1. Providing a detailed description of the instances that will be addressed.
2. Presenting the mathematical formulation used to evaluate the data coming from real-world cellular networks.
3. Report on the best solutions found so far.

CONCLUSIONS:

1. Evaluating the efficacy and efficiency of the newly proposed algorithms with real-world problems instances is of great importance. This deliverable is aimed at setting up the basis for the research on the AFP problem within DIRICOM. This way, it provides data from three real-world cellular networks, the way these data is used to evaluate frequency plans, and the best solutions found so far.

RELATION WITH DELIVERABLES:

PRE: —

CO: —

Real-world instances for the Automatic Frequency Planning problem

DIRICOM

December 2008

1. Introduction

Frequency planning is one of the key problems in the design of GSM systems (Global System for Mobile communications) [9]. In frequency planning, the available frequency band is slotted into channels (or frequencies) which have to be allocated to the elementary transceivers (TRXs) installed in the base stations of the network. This problem is known as Automatic Frequency Planning (AFP), Frequency Assignment Problem (FAP), or Channel Assignment Problem (CAP). AFP is a hard design task because the usable radio spectrum is very scarce and frequencies have to be reused throughout the network and, consequently, some inevitable degree of interference will occur. It is the goal of the designer to minimize the interference and satisfy other design constraints.

Tackling the AFP problem is crucial for today's GSM operators not only at the stage of the initial design, but also in subsequent modifications of the network aimed at solving, for instance, unpredicted interference reports or handling an increase of traffic demand in some areas. Indeed, by mid 2006 GSM services were used by more than 1.8 billion subscribers[1] across 210 countries, representing approximately 77 % of the world's cellular market. It is widely accepted that the third generation mobile telecommunication system (*Universal Mobile Telecommunication System* or UMTS) [11], will coexist with the enhanced releases of the GSM standard (GPRS [4] and EDGE [3]) at least in the first phases. GSM is then expected to play an important role as a dominating technology for many years. Therefore, frequency planning in these networks will be an important task, at present as well as in the future.

Several different problem types are subsumed under the general AFP framework, and many mathematical models have been proposed since the late sixties [1]. This work is focussed on concepts and models which are relevant for current real-world GSM frequency planning [2]. For these reasons, we separate ourselves from existing results, since our problem is far different from those reported in the literature with similar names (which are benchmarking-like problems). We have used a new formulation for the problem [6] so as to take full advantage of realistic and accurate interference information from a real-world GSM network. As a generalization of the graph coloring problem, the FAP is NP-hard [5].

This deliverable focusses on the introduction of three real-world AFP instances coming from GSM networks deployed in three U.S. cities, Seattle, Denver and Los Angeles. The two former ones provide GSM services to over 500,000 users and almost 4 million in the latter case. Solving these instances is therefore of practical interest and it means a direct knowledge transfer to the industry. The next section is devoted to presenting the mathematical formulation adopted for evaluating frequency plans by using the real data coming from the GSM operator. Next, the technical details of the three instances as well as their topology are described. The best known solutions so far are provided next.

1.1. Mathematical Formulation

Let $T = \{t_1, t_2, \ldots, t_n\}$ be a set of n *transceivers*, and let $F_i = \{f_{i1}, \ldots, f_{ik}\} \subset \mathbb{N}$ be the set of valid *frequencies* that can be assigned to a transceiver $t_i \in T$, $i = 1, \ldots, n$. Note that k —the cardinality of F_i— is not necessarily the same for all the transceivers. Furthermore, let $S = \{s_1, s_2, \ldots, s_m\}$ be a set of given *sectors* (or cells) of cardinality m. Each transceiver $t_i \in T$ is installed in exactly one of the m sectors. Henceforth we denote the sector in which a transceiver t_i is installed by $s(t_i) \in S$. Finally, given is a matrix $M = \{(\mu_{ij}, \sigma_{ij})\}_{m\times m}$, called the *interference matrix*. The two elements μ_{ij} and σ_{ij} of a matrix entry $M(i,j) = (\mu_{ij}, \sigma_{ij})$ are numerical values greater or equal than zero. In fact, μ_{ij} represents the mean and σ_{ij} the standard deviation of a Gaussian probability distribution describing the carrier-to-interference ratio (C/I) [13] when sectors i and j operate on a same frequency. The higher the mean value, the lower the interference and thus the better the communication quality. Note that the interference matrix is defined at sector (cell) level, because the transceivers installed in each sector all serve the same area.

A solution to the problem is obtained by assigning to each transceiver $t_i \in T$ one of the frequencies from F_i. A solution (or frequency plan) is henceforth denoted by $p \in F_1 \times F_2 \times \cdots \times F_n$, where $p(t_i) \in F_i$ is the frequency assigned to transceiver t_i. The objective is to find a solution p that minimizes the following cost function:

[1] http://www.wirelessintelligence.com/

$$C(p) = \sum_{t \in T} \sum_{u \in T, u \neq t} C_{\text{sig}}(p,t,u) \ . \tag{1}$$

In order to define the function $C_{\text{sig}}(p,t,u)$, let s_t and s_u be the sectors in which the transceivers t and u are installed, that is, $s_t = s(t)$ and $s_u = s(u)$, respectively. Moreover, let $\mu_{s_t s_u}$ and $\sigma_{s_t s_u}$ be the two elements of the corresponding matrix entry $M(s_t, s_u)$ of the interference matrix with respect to sectors s_t and s_u. Then,

$$C_{\text{sig}}(p,t,u) = \begin{cases} K & \text{if } s_t = s_u, |p(t) - p(u)| < 2 \\ C_{\text{co}}(\mu_{s_t s_u}, \sigma_{s_t s_u}) & \text{if } s_t \neq s_u, \mu_{s_t s_u} > 0, |p(t) - p(u)| = 0 \\ C_{\text{adj}}(\mu_{s_t s_u}, \sigma_{s_t s_u}) & \text{if } s_t \neq s_u, \mu_{s_t s_u} > 0, |p(t) - p(u)| = 1 \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

$K >> 0$ is a very large constant defined by the network designer so as to make it undesirable allocating the same or adjacent frequencies to transceivers serving the same area. Furthermore, function $C_{\text{co}}(\mu, \sigma)$ is defined as follows:

$$C_{\text{co}}(\mu, \sigma) = 100 \left(1{,}0 - Q\left(\frac{c_{\text{SH}} - \mu}{\sigma}\right)\right) \tag{3}$$

where

$$Q(z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx \tag{4}$$

is the tail integral of a Gaussian probability distribution function with zero mean and unit variance, and c_{SH} is a minimum quality signalling threshold. Function Q is widely used in digital communication systems because it characterizes the error probability performance of digital signals [12]. This means that $Q\left(\frac{c_{\text{SH}} - \mu}{\sigma}\right)$ is the probability of the C/I ratio being greater than c_{SH} and, therefore, $C_{\text{co}}(\mu_{s_t s_u}, \sigma_{s_t s_u})$ computes the probability of the C/I ratio in the serving area of sector s_t being below the quality threshold due to the interferences provoked by sector s_u. That is, if this probability is low, the C/I value in the sector s_t is not likely to be degraded by the interfering signal coming from sector s_u and thus the communication quality yielded is high. (Note that this is compliant as to defining a minimization problem.) On the contrary, a high probability —and consequently a high cost— causes the C/I mostly to be below the minimum threshold c_{SH} and thus incurring in low quality communications.

As function Q has no closed form for the integral, it has to be evaluated numerically. For this purpose we use the complementary error function E:

$$Q(z) = \frac{1}{2} E\left(\frac{z}{\sqrt{2}}\right) \tag{5}$$

In [10], a numerical method is presented that allows the value of E to be computed with a fractional error smaller than $1{,}2 \cdot 10^{-7}$. Analogously, function $C_{\text{adj}}(\mu, \sigma)$ is defined as

$$\begin{aligned} C_{\text{adj}}(\mu, \sigma) &= 100\left(1{,}0 - Q\left(\tfrac{c_{\text{SH}} - c_{\text{ACR}} - \mu}{\sigma}\right)\right) \\ &= 100\left(1{,}0 - \tfrac{1}{2} E\left(\tfrac{c_{\text{SH}} - c_{\text{ACR}} - \mu}{\sigma\sqrt{2}}\right)\right) . \end{aligned} \tag{6}$$

The only difference between functions C_{co} and C_{adj} is the additional constant $c_{\text{ACR}} > 0$ (*adjacent channel rejection*) in the definition of function C_{adj}. This hardware specific constant measures the receiver's ability to receive the wanted signal in the presence of an unwanted signal at an adjacent channel. Note that the effect of constant c_{ACR} is that $C_{\text{adj}}(\mu, \sigma) < C_{\text{co}}(\mu, \sigma)$. This makes sense, since using adjacent frequencies (channels) does not provoke such a strong interference as using the same frequencies.

The new feature of this mathematical model is to be found in the definition of the interference matrix information, which is directly imported from real world GSM frequency planning as currently conducted in the industry (and not generated in a computer by sampling random variables). This definition allows not only the computation of high performance frequency plans, but also the prediction of QoS. Indeed, both the definition of the interference matrix and the subsequent computations carried out to obtain the cost values are motivated by real-world GSM networks since they are related to the computation of the BER (Bit Error Rate) performance of Gaussian Minimum Shift Keying (GMSK), the modulation scheme used for GSM [12].

2. AFP instances

This section presents several technical details of the three AFP instances (Seattle, Denver, and Los Angeles), that will be tackled during the DIRICOM lifetime. Table 1 includes, for each instance, the number of TRXs to be assigned a frequency as well as the total number of available frequencies (frequency spectrum). The aim of showing these values is to highlight the level of frequency reusing that is required in real-world GSM networks. The last three rows in the table define the values used in the mathematical formulation of the previous section: K the penalty of using either a cochannel or an adyacent channel within a sector, and the thresholds c_{SH} and c_{ACR}, that respectively show the minimun signal quality and the adyacent channel rejection. Figures 1, 2 and 3 display the topology of the three networks, in which every triangle represents a sector where one or more TRXs operate.

Cuadro 1: Values that characterize the three instances and the constants used in the mathematical formulation.

Instance	Seattle	Denver	Los Angeles
Number of TRXs	970	2.612	41.923
Number of different frequencies	15	18	69
K		100.000	
c_{SH}		6 dB	
c_{ACR}		18 dB	

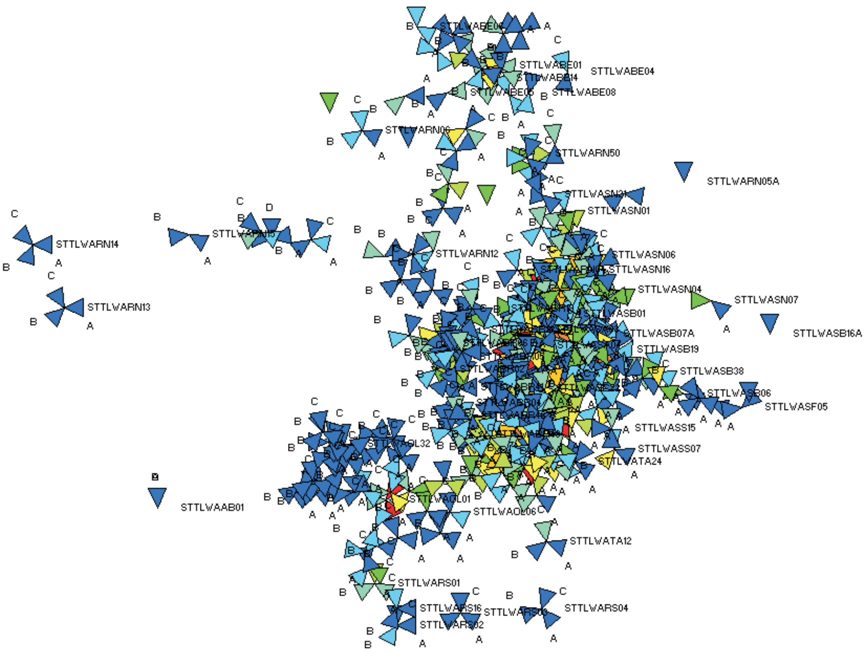

Figura 1: Topology of the Seattle instance.

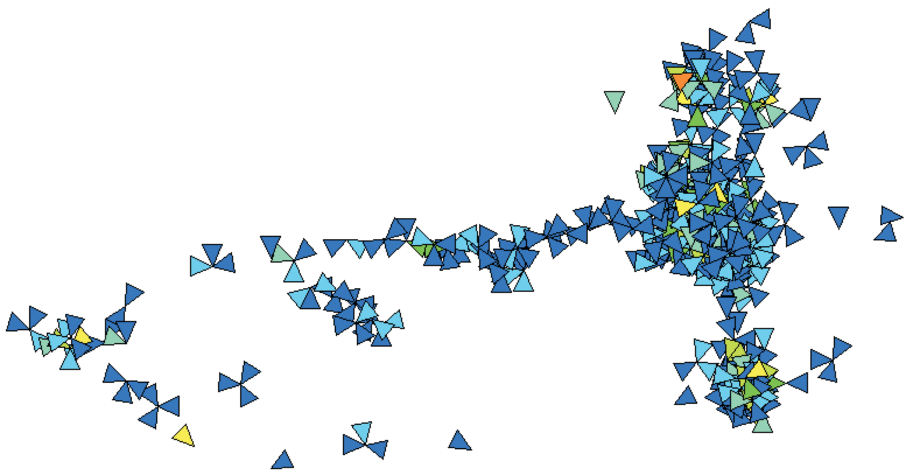

Figura 2: Topology of the Denver instance.

3. Best frequency plans known so far

This section shows the cost of the best frequency plans (provoking the lower interference and thus the lower AFP cost) found up to now for each instances (Table 2). We have also included the algorithm that reached such a solution: a (1 + 1) EA for the Seattle instance [7] and GrEA for both Denver and LA [8].

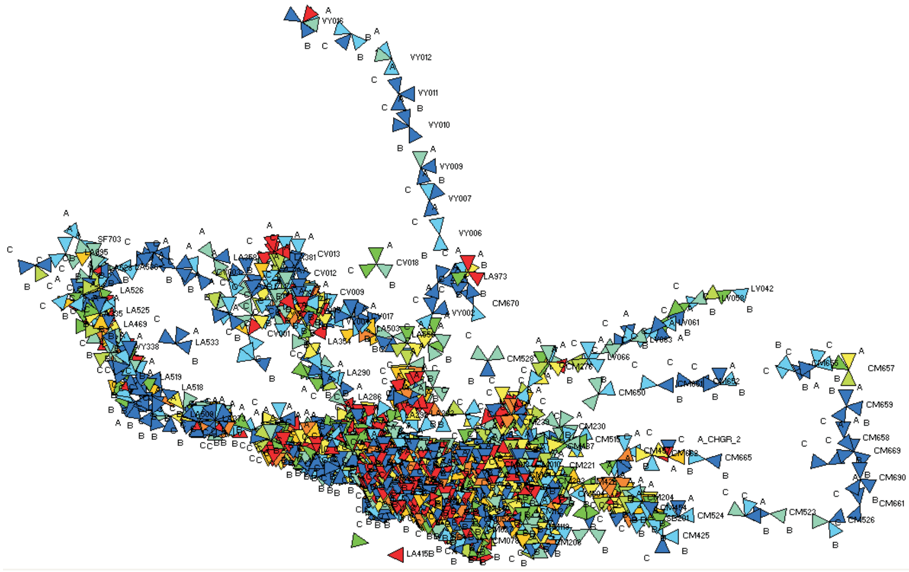

Figura 3: Topology of the LA instance.

Cuadro 2: Best known solutions for the three AFP instances.

Instance	Seattle	Denver	Los Ángeles
Cost	840	83.991	1.012.651
Algorithm	(1+1) EA	GrEA	GrEA

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